Discussion of Signature-based Models of Preventive Maintenance

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Abstract

As a contribution to the discussion of the paper *An overview of some classical models and discussion of the signature-based models of preventive maintenance* (Asadi et al., 2022), we consider the assumption of exchangeability of the failure times of components in systems, which underlies the use of the survival signature for quantification of system reliability. We discuss possible problems for survival signature-based approaches to maintenance planning, in particular where this involves replacement of components.

Keywords: Exchangeability, maintenance, survival signature, system reliability

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1. Survival signature and exchangeability of components' failure times

The survival signature (Coolen and Coolen-Maturi, 2012, 2021a) is a convenient tool for quantification of system reliability. It provides a summary of the system structure which is sufficient for deriving the system failure time distribution given the distributions of the components' failure times. The survival signature exploits assumed exchangeability of failure times of components of the same type, *in fact it is precisely this exchangeability which*

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determines a type of component; we discuss exchangeability in more detail in the next paragraph. The survival signature was introduced by Coolen and Coolen-Maturi (2012) as a generalization to the system signature, which had been presented by Samaniego (2007). The generalization is in the fact that the survival signature is applicable to systems with multiple types of components, while the system signature is only suitable for systems consisting of a single type of components, which was a substantial restriction on its practical use as most systems and networks consist of multiple component types. It is precisely the grouping of components into types which provides advantages for modelling and computation over the use of the system structure function.

An introductory overview to the survival signature was presented by Coolen and Coolen-Maturi (2021b), with emphasis on practical applications. Crucial to this is the assumption of exchangeability of component failure times, hence of the grouping of a system's components into different types. It should be emphasized that the survival signature can be applied without requiring failure times of components of different types to be independent, but, of course, any dependences need to be modelled. Theoretically, the definition of exchangeability of random quantities (de Finetti, 1974) is straightforward, yet often confused with independence. It is perhaps easiest to consider *exchangeability* of failure times of components within a system in the following way: *Suppose that there are multiple components of the same type in a system, which have all functioned for the same period of time. If you get the information that one of these components has failed, then you would have no idea which of these components it is.* It is important that it may *not* be sufficient, or even needed, for components which are defined as being of the same type, to be physically the same components (say that they have the same article number), it is also their functioning in the system, as far as it affects their failure times, that must be considered. So, failure times of components of the same article number are not necessarily exchangeable. On the other hand, failure times of components of different article numbers can be exchangeable, so article numbers and types of components (based on exchangeability) are two different things.

It is important to emphasize that exchangeability of components' failure times in the system is a subjective judgement, and it may just be made to arrive at a suitable level of model detail and complexity. That is to say, one may assume the failure times of components to be exchangeable even though one could study and describe the situation in more detail, possibly leading to a model in which their failure times would be distinguished and no longer be modelled as exchangeable. Decisions on the level of modelling that is adequate for a problem at hand, under practical constraints (e.g. time, budget, information, computer power), are always needed, this is a topic area that deserves more attention from researchers than it has received, see Wooff et al. (2018) for a related discussion of such considerations.

Next, we discuss some aspects of theory for planning of system maintenance, related to the paper by Asadi et al. (2022). While their paper presents results using the system signature and the survival signature separately, we will only discuss from the perspective of survival signature as this generalizes the system signature and indeed coincides with it if all components are of the same type. Asadi et al. (2022) also consider the so-called t-signature, which is related to the system signature but can be used in case of ties between component failure times; as ties are no problem if the survival signature is used, our discussion also applies to scenarios with possible ties.

2. System maintenance

A main consideration for practical systems is the planning of maintenance activities, including inspection of a full system or some of its components. There is a long tradition of research into maintenance modelling and planning in the reliability and operations research literature, to which the paper by Asadi et al. (2022) gives an introduction, while also building on it by implementing the signature-based approaches. Throughout, the crucial point is the exchangeability of the failure times of components of the same type.

Most mathematical models for maintenance activities are based on renewal reward theory, with an assumed repeat of stochastic copies of cycles, allowing relatively simple derivation of costs per unit of time over a very large time horizon. The use of such stochastic copies as cycles requires great care from the perspective of exchangeability of the failure times of components of the same type. For example, a standard age-based preventive replacement model may assume that, if the system has not failed at an inspection time, all components that have failed are replaced while the other components remain in the system. If this happens, and if the components' failure times are not exponentially distributed, then the failure time of a replaced component, at the start of the new cycle, will no longer be exchangeable with components that were of the same type in the previous cycle. This would lead to a new type of component, and if such a system would be used over a long period of time (as implicitly assumed if the renewal reward theory is used), it would eventually lead to all components being of different types, hence the advantages of using the survival signature over the system structure function would disappear.

There is an easy way to avoid this problem, at least from theoretical perspective. One can assume that all components are brought back to an 'as-good-as-new' state at the start of a cycle. For the age replacement problem mentioned above, this may be modelled by a maintenance activity at a cost that differs from the cost for a full replacement of a failed component. There is a, perhaps somewhat surprising, way to maintain exchangeability of component failure times upon maintenance actions, and hence keep the same groupings of components into different types, namely if the number of failed and replaced components is known, but not which specific components in the system they are. This leads to exchangeability of components of the same type still being valid after such replacements, but, crucially, their failure time distributions will now become a mixture of the distribution of the remaining time to failure, for components that had not failed, and the failure time distribution for new components. While this updating of the overall failure time distribution for the exchangeable components of such a type would not be a difficult problem, the resulting cycles, from the perspective of renewal reward theory, would no longer be stochastic copies, hence the costs per unit time cannot be computed anymore using the renewal reward theorem.

The latter scenario above is perhaps surprising in that there are modelling advantages, if one or more components are replaced, to *not* knowing which components these actually are, in order not to have to introduce a new type of components in the survival signature setting. The same holds if one gets more detailed information about the status of specific components by inspection; any such information which leads to a change in the component's remaining time to failure distribution leads to a new component type, but if one has such information without knowing which specific component of the given type it corresponds to, then exchangeability is maintained.

A setting where knowledge of the specific components of a certain type that may fail would not be a problem is up-front planning of availability of spare components (van Houtum and Kranenburg, 2015), as at that stage such knowledge would simply not be available and, due to the exchangeability of failure times of components of the same type, they would all be equally likely to fail. A further opportunity is in more detailed modelling of the state of components, or the system, for which the survival signature has been presented by Qin and Coolen (2022). However, this would also require absence of knowledge of which specific components are in which states, in order to maintain exchangeability of the overall failure time processes.

The possible complication for maintenance modelling and optimisation when combining the use of survival signatures and renewal reward theory could be prevented by changing the optimality criterion, e.g. by considering cost per unit of time over a single cycle rather than over a very large period of time. The use of the one-cycle optimality criterion has been studied in general (Coolen-Schrijner and Coolen, 2006) and also in an adaptive learning scenario, with few modelling assumptions for failure time distributions (Coolen-Schrijner and Coolen, 2007), but not yet in relation to systems with the use of the survival signature.

It is also useful to go beyond the classical maintenance optimisation models, which often were formulated more for theoretical and computational convenience than for their practical benefits, and consider alternative actions that may enhance resilience of systems. One example, presented with the use of survival signatures, is swapping components in a system (Najem and Coolen, 2019). The general message we wish to bring to the reader is that great care is required when using the survival signature for quantification of system reliability in combination with inspection, maintenance and replacement activities, as it is easy to undermine the crucial exchangeability assumption for the failure times of components of the same type, or to end up using the renewal reward theory incorrectly.

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