Survival Signature for Reliability Quantification of Large Systems and Networks

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Abstract. The survival signature is a useful tool for quantification of reliability of large systems and networks with relatively few types of components. This paper provides an introductory overview of the survival signature, with emphasis on recent developments and challenges to enable its use for practical applications. Topics discussed include different survival signatures for specific scenarios, the level of detail in reliability modelling, and computational aspects.
In the literature, system reliability quantification is mostly focused on binary state systems with typically only few components in relatively straightforward configurations and with single functions. Real-world systems, on the other hand, often have multiple levels of functioning and consist of many components in a variety of configurations while they may need to perform multiple functions, leading to substantial challenges for reliability quantification.
In the twelve years since its introduction, the survival signature has gained much attention in the literature, and progress has been made on the challenges indicated above. However, many challenges remain, including some theoretical questions about the very nature of system reliability in real-world situations.

Keywords: Large systems · Survival signature · System reliability · Networks.

1 Survival Signature

Coolen and Coolen-Maturi [6] introduced the survival signature for quantification of reliability of binary state systems with binary state components. Consider a system with $K \geq 1$ types of components, with n_k components of type $k \in \{1, 2, \ldots, K\}$ and $\sum_{k=1}^{K} n_k = n$. The essential assumption is that the random failure times of components of the same type are exchangeable [9, 13]. The state vector $\underline{x} \in \{0, 1\}^n$ of the system describes the states of its components, with 1 representing that a component functions and 0 that it does not function. The system structure function $\phi(\underline{x}) \in \{0, 1\}$ describes the functioning of the system given the component states $\underline{x}$, where 1 represents that the system functions and 0 that it does not function. Due to the arbitrary ordering of the components in the state vector, components of the same type can be grouped together, leading to a state vector that can be written as $\underline{x} = (\underline{x}^1, \underline{x}^2, \ldots, \underline{x}^K)$, with $\underline{x}^k = (x_1^k, x_2^k, \ldots, x_{n_k}^k)$ the sub-vector representing the states of the components of type k.

The *survival signature*, denoted by $\Phi(l_1, l_2, \ldots, l_K)$, with $l_k = 0, 1, \ldots, n_k$ for $k = 1, \ldots, K$, is defined as the probability that the system functions given that *precisely* l_k of its n_k components of type k function, for each $k \in \{1, 2, \ldots, K\}$. There are $\binom{n_k}{l_k}$ state vectors $\underline{x}^k$ with $\sum_{i=1}^{n_k} x_i^k = l_k$; let S_l^k denote the set of these state vectors for components of type k and let $S_{l_1,\ldots,l_K}$ denote the set of all state vectors for the whole system for which $\sum_{i=1}^{n_k} x_i^k = l_k$, $k = 1, 2, \ldots, K$. Due to the exchangeability assumption for the failure times of the n_k components of type k, all the state vectors $\underline{x}^k \in S_l^k$ are equally likely to occur, hence

$$\Phi(l_1, \ldots, l_K) = \left[\prod_{k=1}^{K} \binom{n_k}{l_k}^{-1}\right] \times \sum_{\underline{x} \in S_{l_1,\ldots,l_K}} \phi(\underline{x}) \qquad (1)$$

The survival signature is useful for deriving the probability for the event that the system functions at time $t > 0$, so for $T_S > t$, where T_S is the random system failure time. Let $C_k(t) \in \{0, 1, \ldots, n_k\}$ denote the number of components of type k in the system which function at time $t > 0$, then

$$P(T_S > t) = \sum_{l_1=0}^{n_1} \cdots \sum_{l_K=0}^{n_K} \left\{ \Phi(l_1, \ldots, l_K) P\left(\bigcap_{k=1}^{K} \{C_k(t) = l_k\} \right) \right\} \qquad (2)$$

Equation (2) is the essential result in survival signature theory. It shows that the system survival function can be computed with the required inputs, namely the information about the system structure and about the component failure times, being completely separated. Hence, the effect of changing a system's structure on its survival function can easily be investigated. One can also compare different system structures in general, without assumptions for the random failure times, by comparing the systems' survival signatures [28]. The system survival function is sufficient for important metrics such as the expected failure time of the system, or its remaining time till failure once it has been functioning for some time.

The survival signature requires specification at $\prod_{k=1}^{K}(n_k+1)$ inputs while the structure function must be specified at 2^n inputs; in particular for large values of n and relatively small values of K, so large systems with few component types, the difference is enormous. If all components are of different types, so $K = n$, then the survival signature does not provide any advantages, in the sense of reduced representation, over the structure function. If all components are of the same type, so $K = 1$, then the survival function is closely related to Samaniego's system signature [26, 27].

Equation (2) only requires the assumption that failure times of components of the same type are exchangeable. If one assumes that the failure times of components of different types are independent, then Equation (2) becomes

$$P(T_S > t) = \sum_{l_1=0}^{n_1} \cdots \sum_{l_K=0}^{n_K} \left\{ \Phi(l_1, \ldots, l_K) \prod_{k=1}^{K} P(C_k(t) = l_k) \right\} \tag{3}$$

If, in addition, one assumes that the failure times of components of the same type are independent and identically distributed (*iid*), with known cumulative distribution function (CDF) $F_k(t)$ for type k, then this leads to

$$P(T_S > t) = \sum_{l_1=0}^{n_1} \cdots \sum_{l_K=0}^{n_K} \left\{ \Phi(l_1, \ldots, l_K) \prod_{k=1}^{K} \binom{n_k}{l_k} [F_k(t)]^{n_k - l_k} [1 - F_k(t)]^{l_k} \right\} \tag{4}$$

One can also assume a parametric CDF to enable learning about the parameter based on data, e.g. using Bayesian statistics [2], or use a frequentist statistical method, for example Nonparametric Predictive Inference [10, 11]. The general formula for the system survival function, Equation (2), can also be applied if components' failure times are dependent, for example there may be common-cause failure modes, a risk of cascading failures, load sharing between components and so on. Initial studies into several of such possibilities have been published [7, 15, 16] and there are many related research challenges, in particular on modelling actual dependencies in real-world scenarios.

In the remainder of this paper, a brief overview of the theoretical developments of the survival signature concept is presented, together with a discussion of some key challenges for practical implementation of the concept to challenging real-world scenarios. Section 2 discusses several generalizations of the survival signature while computational aspects are considered in Section 3. Some possibly less obvious issues for practical modelling to quantify reliability of large-scale systems are discussed in Section 4. The paper ends with brief mentioning of some further results and challenges in Section 5.

2 Generalized Survival Signatures

The survival signature concept has been generalized in several ways. A crucial generalization is for multi-state systems, where both the system and components can have multiple states ranging from perfect functioning to failure. Qin and Coolen [23] presented this generalization, enabling a wide variety of applications to be developed, including support for inspection and replacement decisions. Of course, the processes of state changes for components must be modelled and mapping the components' states to the system state can be a complex task, but this is anyhow required if one aims at reliability quantification of multi-state systems.

Coolen-Maturi et al. [12] generalised the concept of the survival signature for multiple systems with multiple types of components and with some components shared between systems. A particularly important feature is that the functioning of these systems can be considered at different times, enabling computation of relevant conditional probabilities with regard to a system's functioning conditional on the status of another system with which it shares components. This is important for many practical systems and networks, for example computer networks, but the theory has a wider relevance as it can directly be applied to a system which performs multiple functions, which is the case for many practical systems. This has led to a substantial area of research, typically considering specific reliability scenarios or restricted system structures, e.g. Yi et al [31] consider systems with a monotone structure function.

Huang et al. [19] presented survival signatures for general phased-missions scenarios, which have several additional challenges such as the possibility that not all components of one type function in the same phases. Whilst such modelling can become rather complex, it does provide a framework for decision support, for example on optimal ordering of phases or re-ordering in case some components are known to have failed during the process.

3 Computational Aspects

For reliability of small systems and networks one can simply derive the system structure function and use Equation (1) to compute the survival signature. This approach has been implemented in the statistical software R [1], and can be used for small to medium-sized systems and networks. Reed [24] presented a substantial improvement on the required computation time by using binary decision diagrams, which can also be used for reliability of multi-terminal networks [25]. Using basic combinatorics, one can compute the survival signature of a system consisting of two subsystems in either series or parallel configuration, if the survival signatures of those subsystems are available; this enables quick computation of the survival signatures of series-parellel systems of any size [11]. As a generalization of this combinatorial method, the survival signature for a multi-state system can be easily derived from the survival signatures of its subsystems if the state of the system is a function of the states of the subsystems [23].

The main reason for the introduction of the survival signature is to enable quantification of system reliability, and related statistical inferences, for large real-world systems and networks, for which one normally would not have the full structure function available. We can think here, for example, about large industrial systems or transportation networks with thousands of components. For such cases, one may need to approximate the survival signature. To do so, it is particularly useful that the survival signature of a coherent system is an increasing function. Approximating the survival signature has received much attention. For example, Behrensdorf et al [4] use percolation theory to exclude areas of the input space of the survival signature where its value does not increase, followed by approximation of the survival signature in the other parts of the input space by Monte Carlo (MC) methods. They illustrate their method on a model of the Great Britain (GB) electricity transmission network, consisting of 29 nodes of two types, and on a model of the Berlin metro network, consisting of 306 nodes and 350 edges, with the nodes divided into two types based on their degree. Also using MC, Di Maio et al [14] use entropy to direct the sampling towards non-trivial areas of the input space, and they illustrate their method on the same GB electricity transmission network. Recently, Lopes da Silva and Sullivan [29] have presented a powerful method to approximate the survival signature for two-terminal networks with two types of components. They show that each MC replication to estimate the survival signature entails solving a multi-objective maximum capacity path problem, and adapt a Dijkstra-like bi-objective shortest path algorithm to solve this problem. They show the efficiency of their algorithm compared to other approaches, which increases with the size of the network, by application to several networks including a power system, which has 4,000 nodes and 29,336 arcs and includes cycles and self-loops.

Once the survival signature of a system or network has been derived, or approximated, it is a useful tool for a range of objectives. For example, it enables very efficient simulation to learn the system survival function, as presented by Patelli et al [22] and extended by George-Williams et al [18] for inclusion of dependent failures. It is also useful for statistical inference for the system reliability, as learning from data, possibly in combination with the use of expert judgements, is crucial in many applications. If one has data available on the individual component types, then inference on the system's failure time is quite straightforward. Nonparametric Predictive Inference [10], a frequentist approach using few modelling assumptions made possible by the use of imprecise probabilities [3], can be used to derive bounds for the system survival function [11]. The application of Bayesian methods has been presented as well [2], this is particularly useful if one has relatively little data on component failures and therefore wishes to include expert judgements. Walter et al [30] generalized the Bayesian approach combined with the survival signature by using sets of priors, as typically done in theory of robust Bayesian methods. They showed that, by choosing the sets of priors in a specific way, one can enable detection of conflict between prior judgements and data, when data become available and are used to update the prior distributions. This can be of great practical importance, as it can point to prior judgements being too optimistic, hence the system reliability may be substantially lower than was originally thought.

4 Modelling for System Reliability

The main challenges for reliability quantification for large systems and networks result from the size and complexity of real-world systems and networks. This includes many factors, such as functional requirements and environmental circumstances, and it may well be that no two components are believed to have exchangeable failure times. Also, one may wish to distinguish many different functioning states for components and systems. However, the key issue in developing a mathematical model for the reliability of a practical system is the target aim of the model, which is typically to support some decision processes. Crucially, there tend to be limited time and resources for the modelling, which will imply that one does not need to, or even can, include all factors that could distinguish between the random failure types of components in the model. Crucially, exchangeability assumptions for the failure times of different components should not be interpreted as strong judgements of their failure time distributions being identical, but instead they are choices with regard to the level of modelling of the system [9]. Similarly, the number of functioning states is typically determined by the corresponding decision problems for which the modelling is undertaken. The main consideration here, from practical perspective, is to choose levels of modelling which are suitable for the tasks whilst being achievable given practical constraints, and not to aim at models which are more detailed representations of reality than is necessary.

An aspect of quantification of system reliability which has received relatively little attention in the literature, but is of great practical importance, is the choice of which components to include in the study. Intuitively, one would consider it necessary to provide a complete model, but for large systems the definition of what components actually are, and which are relevant for describing the reliability of the system for a specific application, has received little attention. From this perspective, Coolen and Coolen-Maturi [8] argued in favour of a change of the nature of the system structure function, from deterministic to stochastic, meaning that the system structure function value for given states of the components is a probability distribution over possible states, rather than a single state. This would enable modelling of system reliability based on only the states of a subset of its components, whilst statistical inference would remain possible based on data from the process. It could also reflect uncertain influences on the system reliability which may not be taken into account explicitly, such as variable environmental circumstances or variations in the use of the system. This is an area where substantial research progress would be needed, which would best be based on practical applications.

5 Concluding remarks

This paper has provided an introductory overview of the survival signature with some discussion of recent developments and main challenges. Many examples of powerful methodology for system reliability quantification enabled by the use of survival signatures have not been discussed, these include new component reliability importance measures [17], resilience achieved by swapping components within a system [21], reliability-redundancy allocation [20], stochastic comparison of different systems [28] and stochastic processes to describe the system reliability over time with varying assumptions on loads or failure processes [5]. It should be noticed that quite some well-known decision processes, in particular management of systems which requires planning of inspections and maintenance activities, fit very well with the level of modelling corresponding to the survival signature. For example, on determining stocks of spare components it is typically not relevant, if there are multiple components of the same type in a system, to consider which specific component may fail at some future time, only its type may be relevant. There may also be opportunities to apply the survival signature concept to scenarios away from traditional engineering, for example the resilience of big organisational structures could be modelled with survival signatures if an organisation has a workforce which mainly consists of groups of workers with exchangeable skills.

Since its introduction, just over a decade ago, the survival signature has received increasing attention from academic researchers and has been acknowledged to be a powerful tool for quantification of reliability of systems and networks. The main intention has always been to provide a practical tool that enables upscaling of reliability quantification, and related statistical inference and decision support, to large scale practical systems and networks, consisting of thousands of components. The route to achieve this still has several major challenges, including further development of computational methods and new ways to model the practically relevant aspects of systems. The achievements of the survival signature, as reported in the literature thus far, are substantial and indicate that a step-change in reliability quantification for large-scale practical systems and networks is feasible.

References

1. Aslett, L.J.M.: ReliabilityTheory: Tools for structural reliability analysis (2012), R package
2. Aslett, L.J.M., Coolen, F.P.A., Wilson, S.P.: Bayesian inference for reliability of systems and networks using the survival signature. Risk Analysis **35**, 1640–1651 (2015)
3. Augustin, T., Coolen, F.P.A., de Cooman, G., Troffaes, M.C.M.: Introduction to Imprecise Probabilities. Wiley, Chichester (2014)
4. Behrensdorf, J., Regenhardt, T.E., Broggi, M., Beer, M.: Numerically efficient computation of the survival signature for the reliability analysis of large networks. Reliability Engineering & System Safety **216**, 107935 (2021)
5. Chang, M., Huang, X., Coolen, F.P.A., Coolen-Maturi, T.: New reliability model for complex systems based on stochastic processes and survival signature. European Journal of Operational Research **309**, 1349–1364 (2023)
6. Coolen, F.P.A., Coolen-Maturi, T.: Generalizing the signature to systems with multiple types of components. In: Zamojski, W., et al. (eds.) Complex Systems and Dependability, pp. 115–130. Springer, Heidelberg (2012)
7. Coolen, F.P.A., Coolen-Maturi, T.: Predictive inference for system reliability after common-cause component failures. Reliability Engineering & System Safety **135**, 27–33 (2015)
8. Coolen, F.P.A., Coolen-Maturi, T.: The structure function for system reliability as predictive (imprecise) probability. Reliability Engineering & System Safety **154**, 180–187 (2016)
9. Coolen, F.P.A., Coolen-Maturi, T.: The survival signature for quantifying system reliability: an introductory overview from practical perspective. In: van Gulijk, C., Zaitseva, E. (eds.) Reliability Engineering and Computational Complexity, pp. 23–37. Springer, Heidelberg (2021)

10. Coolen, F.P.A., Coolen-Maturi, T.: Nonparametric predictive inference. In: Lovric, M. (ed.) International Encyclopedia of Statistical Science (2nd Edition). Springer, Heidelberg (2024), to appear
11. Coolen, F.P.A., Coolen-Maturi, T., Al-nefaiee, A.H.: Nonparametric predictive inference for system reliability using the survival signature. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability **228**, 437–448 (2014)
12. Coolen-Maturi, T., Coolen, F.P.A., Balakrishnan, N.: The joint survival signature of coherent systems with shared components. Reliability Engineering & System Safety **207**, 107350 (2021)
13. De Finetti, B.: Theory of Probability. Wiley, London (1974)
14. Di Maio, F., Pettorossi, C., Zio, E.: Entropy-driven monte carlo simulation method for approximating the survival signature of complex infrastructures. Reliability Engineering & System Safety **231**, 108982 (2023)
15. Eryilmaz, S., Coolen, F.P.A., Coolen-Maturi, T.: Marginal and joint reliability importance based on survival signature. Reliability Engineering & System Safety **172**, 118–128 (2018)
16. Eryilmaz, S., Coolen, F.P.A., Coolen-Maturi, T.: Mean residual life of coherent systems consisting of multiple types of dependent components. Naval Research Logistics **65**, 86–97 (2018)
17. Feng, G., Patelli, E., Beer, M., Coolen, F.P.A.: Imprecise system reliability and component importance based on survival signature. Reliability Engineering & System Safety **150**, 116–125 (2016)
18. George-Williams, H., Feng, G., Coolen, F.P.A., Beer, M., Patelli, E.: Extending the survival signature paradigm to complex systems with non-repairable dependent failures. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability **233**, 505–519 (2019)
19. Huang, X., Aslett, L.J.M., Coolen, F.P.A.: Reliability analysis of general phased mission systems with a new survival signature. Reliability Engineering & System Safety **189**, 416–422 (2019)
20. Huang, X., Coolen, F.P.A., Coolen-Maturi, T.: A heuristic survival signature based approach for reliability-redundancy allocation. Reliability Engineering & System Safety **185**, 511–517 (2019)
21. Najem, A., Coolen, F.P.A.: System reliability and component importance when components can be swapped upon failure. Applied Stochastic Models in Business and Industry **35**, 399–413 (2019)
22. Patelli, E., Feng, G., Coolen, F.P.A., Coolen-Maturi, T.: Simulation methods for system reliability using the survival signature. Reliability Engineering & System Safety **167**, 327–337 (2017)
23. Qin, J., Coolen, F.P.A.: Survival signature for reliability evaluation of a multi-state system with multi-state components. Reliability Engineering and System Safety **218**, 108129 (2022)
24. Reed, S.: An efficient algorithm for exact computation of system and survival signatures using binary decision diagrams. Reliability Engineering & System Safety **165**, 257–267 (2017)
25. Reed, S., Löfstrand, M., Andrews, J.: An efficient algorithm for computing exact system and survival signatures of k-terminal network reliability. Reliability Engineering & System Safety **185**, 429–439 (2019)
26. Samaniego, F.J.: On closure of the ifr class under formation of coherent systems. IEEE Transactions on Reliability **34**, 69–72 (1985)

27. Samaniego, F.J.: System Signatures and their Applications in Engineering Reliability. Springer, New York (2007)
28. Samaniego, F.J., Navarro, J.: On comparing coherent systems with heterogeneous components. Advances in Applied Probability **48**, 88–111 (2016)
29. Lopes da Silva, D.B., Sullivan, K.M.: An optimization-based monte-carlo method for estimating the two-terminal survival signature of networks with two component classes. Preprint (2023), https://industrial-engineering.uark.edu/_resources/TwoTerminal-SurvivalSig-MC.pdf (version 26.05.2023).
30. Walter, G., Aslett, L.J.M., Coolen, F.P.A.: Bayesian nonparametric system reliability using sets of priors. International Journal of Approximate Reasoning **80**, 67–88 (2017)
31. Yi, H., Balakrishnan, N., Li, X.: Joint signatures of two or more semi-coherent systems with shared components. Reliability Engineering & System Safety **242**, 109713 (2024)